Decoding Productivity:
Business Cycle Properties of Labor Productivity Growth

Arturo Estrella*

First draft: September 2003
This revision: June 2004

JEL Codes: O40, E32

Abstract: Among the variables most often tracked by economic analysts and policymakers, the rate of growth of labor productivity is one of the most volatile and hardest to interpret, at least in the short run. Yet, it is often used to explain movements in inflation and output under the assumption that there are stable relationships between productivity growth and those variables. This paper uses frequency domain techniques to separate signal from noise in labor productivity growth and to examine its covariation with other variables over the business cycle. For instance, productivity growth is seen to have strong cyclical connections with unemployment, output growth, and inflation. The frequency domain results clarify these relationships and shed light on several long-standing issues, such as the alleged procyclicality of productivity growth, the Dunlop-Tarshis phenomenon, and the Reder hypothesis.

Presented at NBER/CRIW workshop on productivity, July 26-27, 2004. The views expressed in this paper are those of the author and do not necessarily represent those of the Federal Reserve Bank of New York or the Federal Reserve System.

* Federal Reserve Bank of New York, 33 Liberty Street, New York, NY 10045.
Tel.: 1-212-720-5874. Fax: 1-212-720-1582. E-mail: arturo.estrella@ny.frb.org.

1. Introduction

Among the variables most often tracked by economic analysts and policymakers, the rate of growth of labor productivity is one of the most volatile and hardest to interpret, at least in the short run. At first glance, for example, the productivity series in the United States looks like a combination of a roughly linear trend plus noise, essentially a random walk with a drift. Yet, labor productivity is often used to explain movements in other series with much less noisy behavior – like inflation and output – under the assumption that there are stable relationships between productivity growth and those variables.

This paper uses frequency domain techniques more intensively than the earlier literature to separate signal from noise in labor productivity growth and to uncover consistent covariation with other variables over the business cycle. Results indicate that in fact there is useful information in the low-frequency and business cycle components of productivity growth, but that the information is only clear after a process of decoding in the frequency domain. These techniques may be used to look at either low or business cycle frequency movements, but we focus here on the business cycle components.[1]

Within the business cycle frequencies, we look at movements in productivity relative to output growth and employment growth, which are most often used in the literature as benchmarks to define procyclicality. In addition, cyclical movements in productivity are examined in relation to the unemployment rate, which is less commonly used as a benchmark in

[1] Staiger, Stock and Watson (2001) look at the low frequency relationship between productivity and inflation using a low-pass filter. Time-domain alternatives for estimating a trend component in productivity, based on the Kalman filter, are employed by Gordon (2003), Edge, Laubach and Williams (2004), and Kahn and Rich (2004). Rünstler (2004) uses a time-domain approach to the cyclical component of productivity, based on Harvey (1985).

this literature, but which fits empirically more closely with the NBER definition of the business cycle than output or employment.[2]

After removing the high frequency noise and the low frequency trends, the cyclical component of productivity growth is used to investigate three long-standing issues regarding productivity and real wages. These issues involve various interconnected hypotheses, which are identified in the text. In spite of commonalities among the issues, each one highlights a specific perspective on the behavior of productivity over the cycle.

First, we examine the claim that productivity is procyclical. Real business cycle models predict that productivity is strongly procyclical (for example, moves with employment), whereas Keynesian models predict that it is countercyclical.[3] Earlier empirical work tends to suggest that productivity is negatively correlated with employment over the cycle, though not very strongly, but that its correlation with output is more clearly positive. Focusing on business cycle frequencies helps make sense out of these seemingly disparate results.

The second issue, theoretically connected to the first, is the Dunlop-Tarshis phenomenon. In simple models of production, the level of productivity is closely related to the real wage. Thus, there is a strong connection between the procyclicality of productivity and that of the real wage. Keynes (1936) stated that he was in agreement with the classics as to the countercyclicality of the real wage. However, Dunlop (1938) and Tarshis (1939) soon found contradictory empirical evidence, which has been corroborated in many studies since then. Given the theoretical connection between the real wage and productivity, the cyclical results regarding the latter may be used to infer some properties of the former.

---

[2] See the discussion of Table 1 in Section 2 below.
[3] See, e.g., Christiano and Eichenbaum (1992), Hansen and Wright (1992), Galí (1999), Francis and Ramey (2002).

The third issue is the Reder hypothesis about cyclical changes in the quality of labor. Reder (1955) proposes that "When applicants become scarce, employers tend to lower the minimum standards upon which they insist as a condition for hiring a worker to fill a particular job – and vice versa when applicants become plentiful." In consequence, the quality of labor is lower when the pool of unemployed applicants is relatively limited, as in an economic expansion, and the quality improves when there is more slack in employment, as in a recession. We supplement the Reder hypothesis by proposing that a higher quality labor force is more likely to make better use of technological innovations than one of lower quality, and therefore is more likely to make productivity grow more quickly. This phenomenon is strongly confirmed by the data at business cycle frequencies.

The layout of the paper is as follows. Section 2 discusses the three principal issues in greater detail and considers what type of evidence would be necessary to confirm or refute them empirically. Section 3 presents the statistical methodology of the paper, which is based primarily on analysis of the relevant time series in the frequency domain. More information about the frequency domain techniques is provided in Appendix 2. Section 4 presents the empirical results and considers the weight of the evidence for or against the empirical hypotheses. Conclusions are summarized in Section 5. Appendix 1 describes the data and variables used in the paper.

2. Theoretical background: three issues

2.A. Procyclicality of productivity

There seems to be a broad consensus that productivity is procyclical with regard to the aggregate business cycle, though the exact definition of "procyclical" varies across individual studies. One interpretation is that there is a positive relationship between productivity and output growth. For example, Bernanke and Powell (1986) show that, empirically, the phase lead of U.S. productivity growth with respect to output growth is fairly close to zero at business cycle frequencies, indicating that the variables move roughly contemporaneously. Basu and Fernald (2000) also conclude that productivity is procyclical when they find a positive contemporaneous correlation between productivity growth and output growth. Galí (1999), in a theoretical model, concludes that the contemporaneous correlation between productivity growth and output growth is likely to be positive.

An alternative definition of procyclicality is based on a positive relationship between productivity and employment growth, either unconditionally or conditioning on other economic variables. For instance, the theoretical discussion in Bernanke and Parkinson (1991) looks for a positive partial correlation, after controlling for growth in the capital input to production. The empirical section of that paper focuses on the total correlation between productivity and employment (total hours), which is found to be positive in most industries. Christiano and Eichenbaum (1992) propose a real business cycle model in which there is a positive correlation between productivity and the labor input in response to technology shocks. In the sticky-price model of Galí (1999), in contrast, the positive relationship occurs in response to non-technology shocks.[4] Basu (1996) examines the correlation between productivity growth and the growth in factor inputs more generally and finds that it is positive for U.S. data.

Various reasons have been proposed in the literature for the observed procyclicality of productivity. For example, Bernanke and Parkinson (1991) suggest that it may be a result of true increasing returns to scale or variable labor utilization (labor hoarding), while they reject the influence of technology shocks. Basu (1996), examining a similar set of possible explanations,

[4] A negative relationship results between productivity and labor in the Galí model either unconditionally or in response to technology shocks.

finds in favor of exogenous technology shocks and variable factor utilization. Basu and Fernald (2000), in contrast, downplay technology shocks and suggest that variable factor utilization and resource reallocations are the sources of procyclicality. The European Central Bank (2003) provides evidence of labor hoarding in Europe since 1990. In general, variable factor utilization has received substantial support in the empirical literature, with mixed evidence for other explanations.

In the literature on the procyclicality of productivity, the positive relationship with output is not a theoretical certainty, but does not seem to elicit much surprise. However, the positive relationship with employment is regarded as something of a puzzle. If employment and output are highly correlated, why should there be a distinction? The following simple analysis clarifies this point. Let y be the logarithm of output and n the logarithm of employment or hours worked. The logarithm of productivity (output per hour) may then be expressed as $x = y - n$. Define

$$b_{yn} = \frac{\mathrm{cov}(y,n)}{\mathrm{var}(n)} \tag{1}$$

as the coefficient of a simple regression of output on employment. The covariance between productivity and output is then given by

$$c_{xy} = \mathrm{cov}(y-n, y) = \mathrm{var}(n)\left(\frac{\mathrm{var}(y)}{\mathrm{var}(n)} - b_{yn}\right). \tag{2}$$

Similarly, the covariance between productivity and employment is given by

$$c_{xn} = \mathrm{cov}(y-n, n) = \mathrm{var}(n)\left(b_{yn} - 1\right). \tag{3}$$

Given the variance ratio $v \equiv \mathrm{var}(y)/\mathrm{var}(n)$, the signs of the two covariances (2) and (3) are completely determined by the value of the regression coefficient b_{yn}. Productivity is procyclical with regard to employment if and only if $b_{yn} > 1$, whereas it is procyclical with regard to output if and only if $b_{yn} < v$. Extreme values of b_{yn} thus have opposite implications for c_{xy} and c_{xn}. If b_{yn} is very small, $c_{xy} > 0$ and $c_{xn} < 0$, and these signs are reversed if b_{yn} is very large. If $v > 1$, there is a range of values $1 < b_{yn} < v$ for which both covariances may be positive.[5] It is thus possible – though not certain – that productivity may be procyclical under both the output and employment definitions.

Can the value of b_{yn} be deduced theoretically? Some insights may be obtained from a simple model. Assume a Cobb-Douglas production function in logarithmic form, namely,

$$y = \alpha k + \beta n + \varepsilon, \tag{4}$$

where *k* is the logarithm of the level of capital, ε is a disturbance term orthogonal to the independent variables, and α and β are parameters whose values lie in the unit interval. If returns to scale are constant, then $\alpha + \beta = 1$.

In this framework, we obtain that

$$b_{yn} = \beta + \alpha b_{kn}, \tag{5}$$

where b_{kn} is the coefficient of a regression of *k* on *n*, which is likely to be positive. Thus, as an estimator of β, b_{yn} is biased upwards, which means that b_{yn} may be greater than 1 even if $\beta < 1$ as assumed in the model. Moreover, if we assume that labor and capital are used in fixed proportions and that there are constant returns to scale, then (5) implies that $b_{yn} = 1$ and hence that $c_{xn} = 0$. If $v > 1$, then also $c_{xy} > 0$. Although the assumptions may not be entirely plausible, these values are roughly consistent with the empirical results we encounter later in Section 4.

[5] Empirical estimates using U.S. data generally suggest that $v > 1$. See, e.g., Christiano and Eichenbaum (1992) and Hansen and Wright (1992).

Theory notwithstanding, the procyclicality of productivity, under any definition, remains an empirical question. The analysis of this section suggests the following empirical hypotheses.

Hypothesis 1. Productivity growth is positively correlated with output growth.

Hypothesis 2. Productivity growth is positively correlated with employment growth.

Hypothesis 3. The coefficient of a regression of output growth on employment growth is greater than 1.

Note that equation (3) implies that hypotheses 2 and 3 are equivalent.[6] All three hypotheses are examined empirically in Section 4, with special reference to relationships occurring at business cycle frequencies.

2.B. The Dunlop-Tarshis phenomenon

In “The General Theory,” Keynes (1936, p. 17) says that one point of agreement between him and the classical economists is

> … that, with a given organization, equipment and technique, real wages and the volume of output (and hence of employment) are uniquely correlated, so that, in general, an increase in the employment can only occur to the accompaniment of a decline in the rate of real wages.

The reasoning Keynes (1936) offers for the negative relationship between the real wage and employment rests essentially on the general profit maximization rule $w/p = \partial Y/\partial N$ and on the

[6] By analogy, we could state the additional hypothesis: The coefficient of a regression of output growth on employment growth is less than the ratio of the variances of output and employment growth. By equation (2), this hypothesis is equivalent to Hypothesis 1, but the regression coefficient is compared here with a data-dependent statistic rather than a constant. Thus, it seems preferable to work with the simpler Hypothesis 1 directly.

assumption of diminishing marginal returns to labor, $\partial^2 Y / \partial N^2 < 0$. Here w is the nominal wage, p is the price of output, Y is the level of output, and N is the level of employment.[7]

Very soon after the publication of this statement, Dunlop (1938) and Tarshis (1939) disputed its validity on empirical grounds, producing evidence that suggested that the sign of the relationship went the opposite way. Since then, an extensive literature has ensued in which the hypothesis proposed by Keynes has been empirically challenged time after time.[8]

At first glance, Keynes' statement and the Dunlop-Tarshis reaction may seem unconnected with the cyclicality of productivity. However, economic theory provides a link between the real wage and the level of productivity, which suggests that the issues of section 2.A. are very relevant also in this case. Note also that Keynes, like the literature on the procyclicality of productivity, is ambiguous in the foregoing quote about whether the business cycle is defined in terms of output or employment.

To see the connection between the real wage and productivity, consider a simple neoclassical model with perfect competition. Let the production function be continuous and of the form $Y = f(N) \geq 0$, with $f'(N) > 0$ and $f''(N) < 0$. The function may include other factors as arguments, but we focus only on labor for simplicity.[9] Then, profit maximization implies that there is a function h such that

$$\frac{w}{p} = h\left(\frac{Y}{N}\right),\ h'\left(\frac{Y}{N}\right) > 0. \tag{6}$$

In other words, the real wage varies directly with the level of productivity.[10]

---

[7] See, e.g., Lucas (1970).
[8] A helpful review is Abraham and Haltiwanger (1995).
[9] There is no loss of generality, since the first order condition for profit maximization with respect to labor takes any other factors as fixed.
[10] The main elements of the proof are (1) profit maximization implies that $pf'(N) = w$, (2) $d(w/p)/dN = f''(N) < 0$, and (3) $d(Y/N)/dN = [f'(N)N - f(N)]/N^2 < 0$ for $N > 0$.

The general relationship (6) is easy to see if the production function is Cobb-Douglas or CES. The Cobb-Douglas case is

$$Y = AK^{\alpha} N^{\beta} , \tag{7}$$

where K is the level of capital. Note that this form corresponds to the logarithmic version introduced earlier in equation (4). Profit maximization dictates that

$$w / p = \partial Y / \partial N = \beta Y / N , \tag{8}$$

so that the real wage is a constant multiple of the level of productivity and is therefore perfectly correlated with it. Alternatively, with a constant elasticity of substitution (CES) production function,

$$Y = \gamma \left[\delta K^{-\rho} + (1-\delta) N^{-\rho} \right]^{-1/\rho} , \tag{9}$$

profit maximization implies that

$$w / p = (1-\delta)\gamma^{-\rho} (Y / N)^{1+\rho} . \tag{10}$$

Hence, the real wage is a simple power function of productivity. In logarithmic form, the relationship is linear and again there is perfect correlation. Thus, the countercyclicality of the real wage to which Keynes alludes is very closely tied to the issue of the procyclicality of productivity.

Keynes' theoretical arguments suggest two hypotheses that are in direct opposition to the first two of the previous section.

Hypothesis 4. Productivity growth is negatively correlated with output growth.

Hypothesis 5. Productivity growth is negatively correlated with employment growth.

Section 4 examines these hypotheses in the context of business cycle frequencies.

2.C. The Reder hypothesis

Much of the theory that suggests that productivity is countercyclical assumes that the labor force is homogeneous and that it varies over the business cycle only with regard to the number employed. In Section 2.A., however, we noted that greater modeling flexibility has been achieved by relaxing this assumption. In particular, allowing the level of factor utilization to vary over time has been helpful in explaining some empirical results concerning productivity.

An alternative form of flexibility, proposed by Reder (1955), is that the quality of labor may shift over time, specifically over the course of the business cycle. In economic expansions, when the level of production is growing quickly, there is a need to hire additional workers at an accelerated pace. Since the highest quality workers are presumably hired first, the additional needs are met by reducing the minimum standards applied to new hires. The opposite phenomenon is experienced in economic downturns, when slowing production leads to worker layoffs, with the opportunity to tighten minimum standards and lay off lower quality workers first.

Reder’s theory is driven mainly by cyclical changes in one aspect of the demand for labor. In principle, other cyclical responses in both labor demand and supply could offset the quality effects highlighted by Reder. Labor hoarding, for instance, could imply that changes in quality might be reduced or delayed, mitigating the overall effect. On-the-job learning could also go in the opposite direction, since the average tenure of employed labor may tend to be largest at the end of an expansion. On the supply side, higher real wages in an expansion could attract more skilled labor. The direction of the net effect is thus an empirical question.

To support his theory, Reder (1955) presents empirical evidence using an index of the relative hourly earnings of skilled and unskilled workers. He argues that there is some limited substitutability between lower- and higher-skill workers. In an economic boom, it is possible to upgrade workers to higher-skill levels with limited wage adjustment. The increased relative demand for low-skill workers puts more pressure on their wages, so that the wage differential between skill levels falls. Reder confirms this phenomenon in the context of the two post-war booms in the United States.

More recently, McLaughlin and Bils (2001) provide further evidence that supports the Reder hypothesis, by looking at the cyclical sensitivity of employment and wages in industries with differing wage and skill levels. They find that in high-wage (and therefore high-skill) industries, employment is more cyclical and wages are less procyclical than in lower-wage industries. Both of these results are consistent with Reder's explanation in terms of limited substitutability across skill classes and increased demand for lower skill workers during booms.

In Reder's (1955, 1964) theory, there is a reserve of unemployed workers that may be lured by higher pay during booms. This reserve force arises because minimum wages for low-skill jobs are sticky (at least downward), a phenomenon to which Reder refers as the "social minimum." However, Azariadis (1976) shows that the sticky wage assumption is not necessary for this theory, assuming instead the existence of implicit labor contracts and obtaining similar results.

Reder's reasoning suggests that the rate of unemployment may be a good index of labor quality over the business cycle and that, in examining the cyclical properties of labor productivity, it may be worth taking a look at its relationship with the unemployment rate. Another reason for looking at unemployment in this context is that the U.S. business cycle – as defined by the NBER dating of the cycle – is empirically closely tied to fluctuations in the unemployment rate.

Table 1 shows that the unemployment rate is a better predictor of NBER-defined recessions than either output or hours, which have been traditionally used as benchmarks for the cyclicality of productivity. The table shows some statistics based on probit models of whether or not the U.S. economy is in a recession ($R_t = 1$ versus $R_t = 0$) in quarter *t*. The model based on the unemployment rate alone has a much better fit than models that use output or hours individually. Moreover, the unemployment rate is always highly significant, even if the other variables are included.

Although labor quality should clearly have implications for productivity, the Reder hypothesis in and of itself does not spell these out. We may conjecture that higher quality implies higher productivity, or alternatively that higher quality implies faster productivity growth. The reason for the connection with growth is that higher quality workers are likely to be more adept at taking advantage of technological improvements, leading to faster productivity growth.[11] Using these ideas to complement the strict Reder hypothesis, we thus arrive at the following.

Hypothesis 6. Productivity is positively correlated with the rate of unemployment.

Hypothesis 7. Productivity growth is positively correlated with the rate of unemployment.

These hypotheses are also tested empirically in Section 4 at business cycle frequencies.

3. Empirical methodology

The hypotheses identified in Section 2 have been examined in the earlier literature, mostly using standard time-domain methods, with a mixed range of results in terms of signs and

[11] Reder (1969, page 26) suggests a theoretical connection between the unemployment rate and the level of productivity. Stiroh (2002) shows that trend productivity growth is associated empirically with the intensity of information technology use across industries, which is more consistent with the growth conjecture. We will see in

levels of significance. One of the difficulties in this literature is that productivity growth is very volatile at high frequencies, and this short-term noise tends to obscure any relationships that may exist over the business cycle. The empirical strategy here involves estimating relationships in the frequency domain, where we may focus exclusively on business cycle frequencies while blocking out high frequency noise, as well as trends.

There are some earlier examples of the use of frequency domain techniques to study productivity. For instance, Bernanke and Powell (1986) include some frequency domain methods in their analysis of the cyclical behavior of industrial labor markets in the United States. Among other measures, they examine the coherence and phase lead of several labor market variables, including productivity growth, with respect to output growth. Coherence is averaged over cycles from 1 to 8 years, whereas phase leads are calculated for cycles of 54 months.

A limitation of coherence as a measure of correlation between two variables at a given frequency is that it captures covariation at all lags. Thus, if there is a persistent lag in the relationship, coherence tends to overstate the level of contemporaneous covariation. Of course, looking at the phase lead provides some information in this regard. In looking at the correlation between productivity growth and unemployment, Tripier (2002) employs a more systematic approach, based on "dynamic correlation" as defined by Croux, Forni and Reichlin (2001). Dynamic correlation captures the portion of coherence that corresponds to the in-phase components of the two variables. Tripier defines the U.S. business cycle as cycles of length from 6 quarters to 8 years.[12]

Stock and Watson (1999) employ a technique related to frequency domain filtering to study the covariation with output of a broad range of macroeconomic variables in the United

---

Section 4 that the empirical evidence favors a connection between the level of the unemployment rate and the change in productivity.

States over the business cycle. The business cycle is defined as cycles of length from 6 quarters to 8 years. The time-domain filter they use is developed by Baxter and King (1999) to mimic the effects of frequency domain filtering while avoiding the explicit application of Fourier transforms. Although it is easy to apply and produces results that are generally comparable to frequency domain filters, the Baxter-King filter has a few drawbacks that make it less desirable if frequency domain calculations are feasible.[13]

The empirical methodology used here is intended to exploit frequency domain techniques more thoroughly and consistently than in the foregoing literature. We use most of the statistics found in the earlier literature, but also employ additional measures and techniques that allow for the consistent application of existing statistical theory in the frequency domain.[14] This section contains a brief description of the statistics, and additional technical details are given in Appendix 2.

In contrast to the earlier literature, we represent the business cycle here by means of a reference frequency, which is based on a set of variables that play a central role in the hypotheses of Section 2. These variables are productivity growth, output growth, growth in hours worked, the unemployment rate, and the change in the unemployment rate. The reference frequency is defined as the one that maximizes the minimum bilateral coherence among all pairs in the reference set. The estimated value of the reference frequency is 16 quarters, or 4 years.

---

[12] This definition of the business cycle, though arbitrary, dates at least as far back as Burns and Mitchell (1946).
[13] For instance, the Baxter-King (1999) filter requires dropping several observations at either end of the sample (they recommend 3 years of quarterly data at each end), it implicitly extracts two unit roots (whether or this is necessary), and classical frequency domain statistical theory is not applicable to parameter estimates.
[14] Two useful surveys of statistical theory for the frequency domain, on which this paper draws, are Brillinger (1981) and Koopmans (1995).

In the frequency domain, consistent estimates of spectra and cross spectra at the reference frequency are obtained by averaging with a Daniell window.[15] Like standard frequency domain band pass filters, the Daniell window is an equally-weighted average over a range of frequencies representative of the business cycle. However, in contrast to the usual estimates, which use arbitrary frequencies to define the business cycle, these frequencies are determined by the data and produce consistent estimates of spectra and cross spectra.[16] With the sample size used here, the range of frequencies corresponds approximately to cycles of length between 3 and 7 years.[17]

To illustrate the effects of applying the band pass filter based on the Daniell window, Figure 1 shows the raw productivity growth data (400 times the log first difference in the level of productivity) and its business cycle and low-frequency components from 1954 Q1 to 2003 Q1. The low frequency component is derived from a low pass filter representing cycles longer than the 28-quarter upper bound for the business cycle.

As seen in the dashed lines in Figure 1, quarterly productivity growth is quite volatile and difficult to interpret in real time. However, 85% of the variance of the series is contained in frequency components with cycle length of up to 10 quarters, components whose movements are typically reversed in 3 quarters or less. The spectral decomposition allows "decoding" of the business cycle and low-frequency components, which have much more systematic and persistent patterns over time. As noted earlier, we focus here on business cycle frequencies, which account for 9% of the variance of the series. This component nevertheless exhibits substantial variation over the business cycle, attaining values over a range that extends 5.77 percentage points.

[15] The width of the Daniell window is three-fourths the square root of the number of observations (default in RATS), which provides consistency as well as a good balance between smoothness and definition of spectra and cross spectra. See Koopmans (1995, Section 8.3) and Appendix 2.
[16] The window width rule satisfies the criteria for consistency, e.g., in Brillinger (1981, Section 5.6).
[17] More precisely, between 11 and 28 quarters.

Once the business cycle reference frequency is defined, we compute a series of statistics based on that frequency. First, we look at coherence. As mentioned earlier, coherence is a standard measure of the correlation between two variables at a given frequency, irrespective of phase leads. A high coherence is indicative of high correlation, which may or may not be contemporaneous. The statistical significance of the coherence is measured here by the *t* statistic of the arctanh transformation, as defined in Brillinger (1981, Section 8.7).

As noted earlier, the in-phase correlation focuses on only the in-phase components of two variables at a given frequency.[18] A pair of variables exhibits a high in-phase correlation at a given frequency if the coherence is high and the phase lead is small at that frequency. The squared ratio of the in-phase correlation to the coherence provides an indication of the proportion of the total covariation between the two variables (at a given frequency) that is in-phase or contemporaneous.

Analogously, we can also calculate an in-phase regression coefficient, obtained from a simple regression of the in-phase components of the two variables. The magnitude and the sign of this coefficient are useful in evaluating some of the hypotheses of Section 2. In addition, we can estimate the statistical significance of the in-phase coefficient using the asymptotic distribution given, for instance, in Brillinger (1981, Section 8.5). Note also that the R^2 of this regression is the square of the in-phase correlation.

Finally, we compute the phase lead between pairs of variables at the reference frequency in the standard way.[19] The innovation in this paper is that the phase lead is calculated for the same reference frequency and using the same Daniell window as the other frequency domain

[18] See, e.g., Jenkins and Watts (1968, Section 8.3) or Brillinger (1981, Section 8.4). The in-phase correlation is called "dynamic correlation" by Croux, Forni and Reichlin (2001) and Tripier (2002).
[19] See, e.g., Brillinger (1981, Section 8.4). Sargent (1979, Section XI.6) provides a helpful discussion of phase leads.

statistics, which makes the estimate consistent with all the others and not dependent on an arbitrary definition of the business cycle.[20]

4. Empirical results

This section applies the statistical techniques described above to quarterly U.S. data from 1954 Q1 to 2003 Q1 in order to test the hypotheses identified in Section 2. Although the focus is on productivity and its relationship with the other variables featured in the hypotheses (output growth, employment growth, the unemployment rate), statistics are computed and presented for all pairwise combinations of these variables.[21]

For reference purposes, statistics are also provided for the relationships between each of these variables and both real GDP growth and inflation. These two additional variables are included because they are clearly representative of the U.S. business cycle, and hence their relationships with the variables in the hypotheses can confirm that the results are in fact capturing business cycle phenomena. In interpreting the statistics, we do not focus on GDP growth and inflation, but instead point out a few indicative results. Data definitions and sources are given in Appendix 1.

Table 2 presents pairwise coherences calculated at the reference frequency. Coherences for all pairs are large and significant, as indicated by the *t* statistics, in essence by the definition of the business cycle. The smallest value in the table, the coherence between employment growth

[20] Note that Bernanke and Powell (1986) use a similar frequency of 54 months (4.5 years) to calculate the phase lead. They derive this frequency as the simple average of 1 and 8 years, which define the endpoints of the business cycle in that paper.

[21] Both the level and the change in the unemployment rate are included in the tables. Table 1 shows that the change in highly correlated with the NBER business cycle dating, whereas Section 2 indicates that the level may be more consistent with the Reder hypothesis.

and inflation, is .704 with a t statistic of 4.11. For the basic variables, the lowest coherence is .839 for employment growth and productivity growth, with a t statistic of 5.71.

The uniformly large magnitude of these coherences is consistent with the concept of a business cycle, that is, the comovement of a large number of important macroeconomic variables at a certain range of frequencies, possibly with leads and lags. The coherences are all large also because they abstract from the size of these lags, which we will see may vary substantially across variable pairs. Although the coherence results are helpful in validating the existence of a business cycle, they are too general to be useful in the direct evaluation of the hypotheses of Section 2. However, we have at our disposal a series of related measures, described in the previous section, which may be used to test the hypotheses more precisely.

The squared coherence is the sum of two terms that correspond to in-phase and out-of-phase covariation. If two variables have a zero phase lead at a given frequency, they are perfectly correlated at that frequency and the squared coherence is completely accounted for by the in-phase term. The same is true if the phase lead is π, half a cycle, although in that case the in-phase correlation is perfectly negative. If the phase lead is $\pi / 2$, a quarter of a cycle, the in-phase correlation between the two variables at that frequency is zero and the coherence is dominated by the out-of-phase terms.

Table 3 contains in-phase correlations for the same variable pairs as Table 2. Like the coherences, many of these values are large in absolute value, for example the in-phase correlation between the change in output and the change in employment, or between the change in employment and real GDP growth. However, the table also contains some very small values that indicate that those particular combinations are largely out of phase, such as the change in productivity and the change in employment.

Consider the in-phase correlations of productivity with other variables. The most positive value is obtained for the unemployment rate, the most negative for inflation. In these cases, the in-phase component accounts for at least three quarters of the squared coherence. Less remarkable results are obtained for output growth and employment growth, which we have seen are traditionally used to define procyclicality and are the focus of Hypotheses 1-5 of Section 2. The in-phase correlation of productivity and output is under 50%, and the in-phase correlation with employment is only .055, one of the smallest in the table.

Hypothesis 6 – regarding the levels of productivity and the unemployment rate – is tested in Table 3 in an equivalent form by looking at changes in both of these variables. The corresponding in-phase correlation is quite small and negative, failing to confirm the conjectured positive relationship. In contrast, the in-phase correlation between the unemployment rate and the change in productivity (.805) lends strong preliminary support to Hypothesis 7, which is based on the Reder hypothesis.[22] The statistical significance of these correlations is considered below in the context of the in-phase regressions.

The strong negative relationship between productivity and inflation is consistent with earlier empirical results.[23] It has been attributed, among other reasons, to distortions in work incentives, investment, business operations, and taxes caused by high inflation or uncertainty about inflation.

Looking at other combinations in the table, it is not surprising (though nonetheless reassuring) that pairs that are definitionally close are highly correlated. These pairs include output and GDP growth (.992), and hours and unemployment (-.977). Moreover, all cross

[22] Lynch and Nickell (2001) find a negative relationship between the trend components of productivity growth and the unemployment rate. In spite of the strong positive correlation observed here at business cycle frequencies, that finding is consistent with our frequency domain analysis. For instance, if we focus on low frequency movements (cycles of length at least 28 quarters), the correlation is -.304.

correlations among these variables are relatively high, whether they represent output or employment.

Additional evidence with regard to the hypotheses of Section 2 is obtained from in-phase regressions. These results are found in Table 4, whose format is largely similar to the previous two tables. However, in contrast to correlation, regression coefficients are not symmetric in terms of the two variables, so results are presented with each variable in the pair as the dependent variable, as listed in the first column.

Hypotheses 1 and 4 pertain to the regression of productivity on output, specifically Δx on Δy. The estimated coefficient in this regression is .21, with a *t* statistic of 1.82. This positive point estimate supports the procyclicality of productivity with respect to output, as in Hypothesis 1, and the estimate is significant in a two-sided test at the 10% level, though not at the 5% level (the *p* value is .069). Of course, Hypothesis 4, which is the opposite of Hypothesis 1, is rejected under the same conditions.

Hypotheses 2 and 5 relate to the regression of productivity on employment, that is, Δx on Δn. As in the case of output, the estimated coefficient in this regression is positive. However, the point estimate here is only .03 with a standard error about 5 times as large. Although the point estimate is consistent with Hypothesis 2, there is no compelling support for or against either hypothesis.

Hypothesis 3 refers not just to the sign, but to the magnitude of the coefficient in the regression of output on employment, or Δy on Δn, which is expected to be larger than 1. Because of linear relationships, this result parallels the one regarding productivity and

[23] See, e.g., Clark (1982), Jarrett and Selody (1982), Ram (1984) and Cameron, Hum and Simpson (1996).

employment. The point estimate of the coefficient is in the expected range, but only barely and insignificantly so. In fact, the lower bound of a 95% confidence interval is .74.

The strongest result in connection with any of the hypotheses is from the regression of productivity growth on the unemployment rate (Δx on u), which corresponds to Hypothesis 7 and the Reder hypothesis. The point estimate is 1.39 and the standard error is .31, so that the t statistic is a very significant 4.50. In contrast, there is no support for Hypothesis 6, which posits a positive relationship between the changes in productivity and unemployment.

In Table 5, we turn to our last measure of cyclical comovement, namely the phase leads between pairs of variables at the reference frequency. Since Table 2 shows that all coherences are high, the information derived from phase leads should be similar to the information contained in the in-phase regression results. However, looking at the phase leads explicitly can shed further light on the relationships involving productivity and output, employment or unemployment.

Recall that the components of two variables at a given frequency are highly positively correlated if the phase lead is zero, negatively if the phase lead is half a cycle, and not correlated if the phase lead is one quarter of the cycle. Since the reference cycle is 16 quarters, we can use 0, 8 and 4 quarters, respectively, as benchmarks for these various correlation levels.

The key result from Table 5 for Hypotheses 1 and 4 is the lead between productivity and output. We see from the table that Δx leads Δy by 2.60 quarters (a negative lead is a lag), which is more than halfway toward the benchmark of a quarter of a cycle. This lead helps explain why the corresponding regression coefficient, even though positive, is only marginally significant. Note that Basu, Fernald and Kimball (2002) obtain a similar lagged effect when looking at the response of output with respect to a technology-improvement shock. In their model, the impulse response of output is significant (and positive) only at lags of 2 and 3 quarters.

For Hypotheses 2 and 5, we look at the lead between productivity and employment, and find that $\triangle x$ leads $\triangle n$ by 3.95 quarters, which is very close to the quarter-of-a-cycle benchmark. Again we see an explanation for the earlier in-phase regression results. Productivity and employment are highly correlated as far as coherence is concerned, but the lead of almost one-quarter of a cycle makes them seem uncorrelated when observed contemporaneously. Again, Basu, Fernald and Kimball (2002) obtain a similar lagged effect, but in this case it is preceded by a significantly negative contemporaneous effect, which is not seen here. Note, however, that their identification assumptions with regard to employment are challenged by Christiano, Eichenbaum and Vigfusson (2004), who find a positive but insignificant contemporaneous response in per capita hours and significantly positive responses at lags from 1 to 3 quarters. Those results are more in line with Tables 4 and 5 here, though the lags are somewhat shorter.

Hypothesis 3 is related to the lead between output and employment, and we see that $\triangle y$ leads $\triangle n$ by about one quarter. This means that contemporaneous or in-phase correlation is high, although the lead of 1.01 is statistically significant, with a t statistic of 2.37.

Finally, for Hypothesis 7 we observe that the unemployment rate u leads productivity growth $\triangle x$ by less that one quarter (.88), and that the standard error of 1.08 suggests that this lead is not significantly different from zero. Thus, productivity growth and unemployment are highly correlated contemporaneously, as the in-phase regression results had already shown. Productivity growth is therefore highly countercyclical in an important sense: it moves over the business cycle very closely with unemployment, and we have seen that the change in unemployment is an excellent indicator of NBER dating of recessions in the United States.[24]

[24] The change in unemployment leads the change in productivity by about a quarter of a cycle, suggesting that the levels of unemployment and productivity are almost completely out of phase.

The lead-lag relationships between productivity and unemployment, output or hours are seen graphically in Figure 2. The solid line in each panel of the figure is the business cycle component of productivity growth.[25] The dashed lines in the three panels represent the business cycle components of the unemployment rate, output growth, and hours growth, respectively. The high coherence between the pairs of variables is immediately obvious from the figure, as are the essentially contemporaneous timing in the upper panel and the lags of about 3 and 4 quarters in the other two.

5. Conclusions

Through the use of frequency domain techniques, the results of Section 4 give a more detailed view of the covariation between productivity growth and other cyclical variables over the business cycle than is possible in the time domain. What are the implications of these results for the three larger issues considered in Section 2?

First, there is only limited support for the procyclicality of productivity at business cycle frequencies. The coherence of productivity with the traditional benchmarks of output, employment, and hours worked is high. However, the contemporaneous covariation with these variables is weak or non-existent. The relationship with output indicates a limited degree of procyclicality, significant at the 10% level. However, there is no indication of covariation with employment at these frequencies, since the two variables seem almost exactly out of phase. Finally, productivity growth is found to be countercyclical in an important sense: it is closely correlated with the unemployment rate at business cycle frequencies, as their movements are almost exactly in synch.

[25] The business cycle is defined as cycles of length from 11 to 28 quarters, which as noted earlier is determined by the width of the Daniell window centered at the reference frequency. The business cycle component is the inverse

Second, there is no support at business cycle frequencies for Keynes' contention that real wages should be countercyclical with respect to output or employment. The positive relationship between productivity and output is a somewhat weak confirmation of the Dunlop-Tarshis phenomenon. Although the lack of a relationship between productivity and employment is not strictly a rejection, neither does it support Keynes' claim.

Finally, there is strong support for the Reder hypothesis that the quality of labor varies directly with the unemployment rate. The evidence shows that productivity growth is highly correlated with the unemployment rate and that the relationship is essentially contemporaneous.

Appendix 1. Description of data and variables

The data on productivity, output, employment and unemployment are drawn from the U.S. Department of Labor, Bureau of Labor Statistics. These include non-farm output and non-farm hours worked, whose ratio is the level of productivity, and the civilian unemployment rate for workers age 16 and above. Real GDP and the GDP deflator, both chain-weighted, are from the U.S. Department of Commerce, Bureau of Economic Analysis. The unemployment rate is converted to a quarterly frequency by taking arithmetic averages of monthly data. All variables are seasonally adjusted.

Appendix 2. Details of frequency domain calculations

Let x be a vector containing T observations of a stationary zero-mean time series x_t, $t = 1,...,T$. The frequency domain representation of this series is given by the Fourier transform

$$\tilde{x}(\lambda_k) = \frac{1}{\sqrt{2\pi T}} \sum_{t=1}^{T} x_t e^{-i\lambda_k t} , \tag{11}$$

Fourier transform of the filtered Fourier transform of each variable, retaining only frequencies in the specified range.

where $\lambda_k = 2\pi k / T$, $k = 0,...,T-1$. The function $\tilde{x}(\lambda)$ may be calculated for any $0 \le \lambda < 2\pi$, but it is completely determined by its values at the T frequencies λ_k .[26] A consistent estimate of the spectrum of x_t at the frequency λ_k is computed as

$$\hat{f}_{xx}(\lambda_k) = \frac{1}{2h+1} \sum_{j=k-h}^{k+h} \left| \tilde{x}(\lambda_j) \right|^2 , \tag{12}$$

where $2h+1$ is the odd integer nearest to $3/4\sqrt{T}$. Similarly, if y is another time series defined over the same period, the cross spectrum of x and y may be consistently estimated as

$$\hat{f}_{xy}(\lambda_k) = \frac{1}{2h+1} \sum_{j=k-h}^{k+h} \tilde{x}(\lambda_j) \tilde{y}^{\dagger}(\lambda_j) , \tag{13}$$

where † denotes the complex conjugate.

The <u>coherence</u> between x and y is defined as $\left| R_{xy}(\lambda_k) \right| = \left| \hat{f}_{xy}(\lambda_k) \right| \Big/ \left(\hat{f}_{xx}(\lambda_k) \hat{f}_{yy}(\lambda_k) \right)^{1/2}$.

As noted in the text, the business cycle <u>reference frequency</u> is defined as the λ_k , $k \in \{0,...,T-1\}$, for which the smallest coherence between any two basic reference variables is maximized. If the cross spectrum is expressed in polar form as $\hat{f}_{xy}(\lambda_k) = \left| \hat{f}_{xy}(\lambda_k) \right| e^{i\vartheta(\lambda_k)}$, the <u>phase lead</u> of x over y is $\vartheta(\lambda_k)$. The measurement unit of the phase lead may be converted from radians to periods (quarters) by computing $-\vartheta(\lambda_k)/\lambda_k$.

The <u>in-phase regression coefficient</u> of y on x is calculated as $b_{yx}(\lambda_k) = \mathrm{Re}\left(\hat{f}_{yx}(\lambda_k) \right) \Big/ \hat{f}_{xx}(\lambda_k)$ and the <u>in-phase correlation</u> as $\mathrm{Re}\left(R_{xy}(\lambda_k) \right) = \mathrm{Re}\left(\hat{f}_{xy}(\lambda_k) \right) \Big/ \left(\hat{f}_{xx}(\lambda_k) \hat{f}_{yy}(\lambda_k) \right)^{1/2}$.[27] Note that

[26] See the frequency domain sampling theorem in Koopmans (1995, Section 8.2).
[27] See Brillinger (1981, Section 8.4) and also Jenkins and Watts (1968, Section 8.3).

$$\left|R_{xy}(\lambda_k)\right|^2 = \mathrm{Re}\left(R_{xy}(\lambda_k)\right)^2 + \mathrm{Im}\left(R_{xy}(\lambda_k)\right)^2 \geq \mathrm{Re}\left(R_{xy}(\lambda_k)\right)^2 . \tag{14}$$

Standard errors of the estimates are computed using the asymptotic moments given in Brillinger (1981, Section 8.7). The asymptotic variance of the in-phase regression coefficient estimate is $\left[1-\mathrm{Re}\left(R_{xy}(\lambda_k)\right)^2\right]\hat{f}_{yy}(\lambda_k)\Big/\left[(2h+1)\,\hat{f}_{xx}(\lambda_k)\right]$ and the asymptotic variance of the phase lead estimate is $\left[\left|R_{xy}(\lambda_k)\right|^{-2}-1\right]\Big/\left[2h+1\right]$. For the coherence, we transform the estimate by applying the function $\tanh^{-1}\left|R_{xy}(\lambda_k)\right|$, which has a normal asymptotic distribution with variance $1\big/\left[2(2h+1)\right]$. Brillinger (1981, pages 311-312) suggests that for asymptotic calculations it is preferable to use this transformation, rather than the coherence itself. Note that all of these asymptotic variances apply only to the case of the Daniell window.

The inverse Fourier transform is used in Section 4 to compute the business cycle component of a variable in the time domain. The inverse transform of z_k, $k=0,\ldots,T-1$, is defined as

$$\tilde{z}_t^{-1} = \sqrt{\frac{2\pi}{T}}\sum_{k=0}^{T-1} z_k e^{i\lambda_k t} , \tag{15}$$

where $\lambda_k = 2\pi k/T$, $k=0,\ldots,T-1$ as before. To compute the business cycle component for a variable x_t with Fourier transform $z_k = \tilde{x}(\lambda_k)$, $k=0,\ldots,T-1$, we set $z_k = 0$ for λ_k outside the range of business cycle frequencies and take the inverse transform.

References

Abraham, Kathatrine G. and Haltiwanger, John C. "Real Wages and the Business Cycle." Journal of Economic Literature 33, 1215-1264, September 1995.

Azariadis, Costas. "On the Incidence of Unemployment." Review of Economic Studies 43(1), 115-125, February 1976.

Basu, Susanto. "Procyclical Productivity: Increasing Returns or Cyclical Utilization?" Quarterly Journal of Economics 111(3), 719-751, August 1996.

Basu, Susanto and Fernald, John. "Why Is Productivity Procyclical?" National Bureau of Economic Research Working Paper No. 7940, October 2000.

Basu, Susanto, Fernald, John and Kimball, Miles. "Are Technology Improvements Contractionary?" Harvard Institute of Economic Research Discussion Paper No. 1986, December 2002.

Baxter, Marianne and King, Robert G. "Measuring Business Cycles: Approximate Band-Pass Filters for Economic Time Series." Review of Economics and Statistics 81(4), 575-593, November 1999.

Bernanke, Ben S. and Parkinson, Martin L. "Procyclical Labor Productivity and Competing Theories of the Business Cycle: Some Evidence from Interwar U.S. Manufacturing Industries." Journal of Political Economy 99(3), 439-459, June 1991.

Bernanke, Ben S. and Powell, James L. "The Cyclical Behavior of Industrial Labor Markets: A Comparison of the Prewar and Postwar Eras." In Gordon, Robert J., ed., "The American Business Cycle: Continuity and Change." Chicago IL: University of Chicago Press, 583-621, 1986.

Brillinger, David R. "Time Series Analysis." San Francisco CA: Holden-Day, 1981.

Burns, Arthur M. and Mitchell, Wesley C. “Measuring Business Cycles.” New York NY: National Bureau of Economic Research, 1946.

Cameron, Norman, Hum, Derek and Simpson, Wayne. “Stylized Facts and Stylized Illusions: Inflation and Productivity Revisited.” Canadian Journal of Economics 29(1), 152-162, February 1996.

Christiano, Lawrence J. and Eichenbaum, Martin. “Current Real-Business Cycle Theories and Aggregate Labor Market Fluctuation.” American Economic Review 82(3), June 1992.

Christiano, Lawrence J., Eichenbaum, Martin and Vigfusson, Robert. “The Response of Hours to a Technology Shock: Evidence Based on Direct Measures of Technology.” National Bureau of Economic Research Working Paper No. 10254, January 2004.

Clark, Peter K. “Inflation and the Productivity Decline.” American Economic Review 72(2), 149-154, May 1982.

Croux, Christophe, Forni, Mario and Reichlin, Lucrezia. “A Measure of Comovement for Economic Variables: Theory and Empirics.” Review of Economics and Statistics 83(2), 232-241, May 2001.

Dunlop, John T. “The Movement of Real and Money Wage Rates.” Economic Journal 48(191), 413-434, September 1938.

Edge, Rochelle M., Laubach, Thomas and Williams, John C. “Learning and Shifts in Long-Run Productivity Growth.” Federal Reserve Bank of San Francisco Working Paper No. 2004-04, March 2004.

Engle, Robert F. “Band Spectrum Regression.” International Economic Review 15(1), 1-11, February 1974.

Estrella, Arturo. “A New Measure of Fit for Equations with Dichotomous Dependent Variables.” Journal of Business and Economic Statistics 16(2), 198-205, April 1998.

European Central Bank. “Monthly Bulletin.” Frankfurt am Main, 30-31, July 2003.

Francis, Neville and Ramey, Valerie. “Is the Technology-Driven Real Business Cycle Hypothesis Dead? Shocks and Aggregate Fluctuations Revisited.” National Bureau of Economic Research Working Paper No. 8726, January 2002.

Galí, Jordi. “Technology, Employment, and the Business Cycle: Do Technology Shocks Explain Aggregate Fluctuations?” American Economic Review 89(1), 249-271, March 1999.

Gordon, Robert J. “Exploding Productivity Growth: Context, Causes, and Implications.” Brookings Papers on Economic Activity, 207-298, Issue 2, 2003.

Hansen, Gary D. and Wright, Randall. “The Labor Market in Real Business Cycle Theory.” Federal Reserve Bank of Minneapolis Quarterly Review, Spring 1992.

Harvey, A.C. “Trends and Cycles in Macroeconomic Time Series.” Journal of Business and Economic Statistics 3(3), 216-227, 1985.

Jarrett, J. Peter and Selody, Jack G. “The Productivity-Inflation Nexus in Canada, 1963-1979.” Review of Economics and Statistics 64(3), 361-367, August 1982.

Jenkins, Gwilym M. and Watts, Donald G. “Spectral Analysis and Its Applications.” San Francisco CA: Holden-Day, 1968.

Kahn, James A. and Rich, Robert W. “Tracking the New Economy: Using Growth Theory to Detect Changes in Trend Productivity.” Federal Reserve Bank of New York Working Paper, March 2004.

Keynes, John Maynard. “The General Theory of Employment, Interest, and Money.” New York NY: Harbinger, 1964 (1936).

Koopmans, Lambert H. "The Spectral Analysis of Time Series." San Diego CA: Academic Press, 1995.

Lucas, Robert E., Jr. "Capacity, Overtime, and Empirical Production Functions." American Economic Review 60(2), 23-27, May 1970.

Lynch, Lisa M. and Nickell, Stephen. "Rising Productivity and Falling Unemployment: Can the U.S. Experience Be Sustained and Replicated?" In Krueger, Alan B. and Solow, Robert M., eds., "The Roaring Nineties: Can Full Employment Be Sustained?" New York: Russell Sage Foundation, Century Foundation Press, 538-577, 2001.

McLaughlin, Kenneth J. and Bils, Mark. "Interindustry Mobility and the Cyclical Upgrading of Labor." Journal of Labor Economics 19(1), 94-135, January 2001.

Ram, Rati. "Causal Ordering Across Inflation and Productivity Growth in the Post-War United States." Review of Economics and Statistics 66(3), 472-477, August 1984.

Reder, M. W. "The Theory of Occupational Wage Differentials." American Economic Review 45(5), 833-852, December 1955.

Reder, M. W. "Wage Structure and Structural Unemployment." Review of Economic Studies 31(4), 309-322, October 1964.

Reder, M. W. "The Theory of Frictional Unemployment." Economica 36(141), 1-28, February 1969.

Rünstler, Gerhard. "Modelling Phase Shifts among Stochastic Cycles." Econometrics Journal 7, 232-248, 2004.

Sargent, Thomas J. "Macroeconomic theory." New York NY: Academic Press, 1979.

Staiger, Douglas, Stock, James H. and Watson, Mark W. "Prices, Wages, and the U.S. NAIRU in the 1990s." In Krueger, Alan B. and Solow, Robert M., eds., "The Roaring Nineties: Can

Full Employment Be Sustained?" New York: Russell Sage Foundation, Century Foundation Press, 3-60, 2001.

Stiroh, Kevin J. "Information Technology and the U.S. Productivity Revival: What Do the Industry Data Say?" American Economic Review 92, 1559-1576.

Stock, James H. and Watson, Mark W. "Business Cycle Fluctuations in U.S. Macroeconomic Time Series." In Taylor, John B. and Woodford, Michael, eds., "Handbook of Macroeconomics." Amsterdam: North-Holland, 1999.

Tarshis, Lorie. "Changes in Real and Money Wage Rates." Economic Journal 49(193), 150-154, March 1939.

Tripier, Fabien. "The Dynamic Correlation between Growth and Unemployment." Economics Bulletin 5(4), 1-9, 2002.

Table 1. Reference variables and NBER cycles: p values from likelihood ratio exclusion tests and probit R^2 s
1954 Q1 to 2000 Q4

$$P(R_t = 1) = \Phi(\alpha_0 + \alpha_1 \Delta u + \alpha_2 \Delta y + \alpha_3 \Delta n)$$

Equation	p values			R^2
	Δu	Δy	Δn	
1	.000			.706
2		.000		.374
3			.000	.571
4	.000	.027		.742
5		.007	.000	.618
6	.000		.287	.714
7	.000	.040	.495	.745

Notes: $y =$ log of output, $n =$ log of hours worked, and $u =$ unemployment rate. $R_t = 1$ if t is a recession quarter (after an NBER peak up to the following NBER trough) and $R_t = 0$ otherwise. Φ is the standard normal cumulative distribution function and R^2 is calculated as in Estrella (1998).

Table 2. Coherences
1954 Q1 to 2003 Q1

Variable 1	Variable 2 $\triangle x$	$\triangle y$	$\triangle n$	u	$\triangle u$
$\triangle y$	.878 (6.42)	–			
$\triangle n$	.839 (5.71)	.972 (9.99)	–		
u	.852 (5.93)	.947 (8.44)	.941 (8.20)	–	
$\triangle u$	.848 (5.86)	.960 (9.15)	.977 (10.48)	.946 (8.39)	–
$\triangle g$	.867 (6.20)	.994 (13.47)	.967 (9.62)	.942 (8.22)	.950 (8.60)
$\triangle p$	.810 (5.28)	.754 (4.61)	.704 (4.11)	.821 (5.44)	.715 (4.20)

Notes: $y =$ log of output, $n =$ log of hours worked, $x =$ log of labor productivity ($= y - n$), $u =$ unemployment rate, g = log of real GDP, $p =$ log of GDP deflator. Cycle is 16.5 quarters, number of frequency domain ordinates is 197, width of spectral smoothing window is 11. Figures in parentheses are t statistics, which were obtained by applying the arctanh transformation, as suggested in Brillinger (1981, Section 8.7).

Table 3. In-phase correlation and proportion of squared coherence attributable to in-phase component
1954 Q1 to 2003 Q1

Variable 1	Variable 2 Δx	Δy	Δn	u	Δu
Δy	.481 (.300)	–			
Δn	.055 (.004)	.902 (.860)	–		
u	.805 (.892)	.180 (.036)	-.192 (.042)	–	
Δu	-.044 (.003)	-.876 (.833)	-.977 (.999)	.224 (.056)	–
Δg	.429 (.245)	.992 (.996)	.918 (.901)	.114 (.015)	-.889 (.876)
Δp	-.696 (.740)	-.689 (.835)	-.440 (.390)	-.517 (.397)	.416 (.339)

Notes: $y =$ log of output, $n =$ log of hours worked, $x =$ log of labor productivity ($= y - n$), $u =$ unemployment rate, $g =$ log of real GDP, $p =$ log of GDP deflator. Cycle is 16.5 quarters, number of frequency domain ordinates is 197, width of spectral smoothing window is 11. See Jenkins and Watts (1968, Section 8.3), Brillinger (1981, Section 8.4).

Table 4. In-phase regression coefficients
1954 Q1 to 2003 Q1

Dependent Variable	Regressor: Δx	Δy	Δn	u	Δu
Δx	–	.21 (.11)	.03 (.15)	1.39 (.31)	-.22 (1.52)
Δy	1.12 (.61)	–	1.03 (.15)	.72 (1.19)	-10.24 (1.70)
Δn	.11 (.61)	.79 (.11)	–	-.67 (1.04)	-10.02 (.66)
u	.47 (.10)	.04 (.07)	-.05 (.08)	–	.66 (.86)
Δu	-.01 (.06)	-.07 (.01)	-.10 (.01)	.08 (.10)	–
Δg	.74 (.47)	.74 (.03)	.78 (.10)	.34 (.89)	-7.76 (1.20)
Δp	-.56 (.17)	-.24 (.08)	-.17 (.11)	-.72 (.36)	1.69 (1.11)

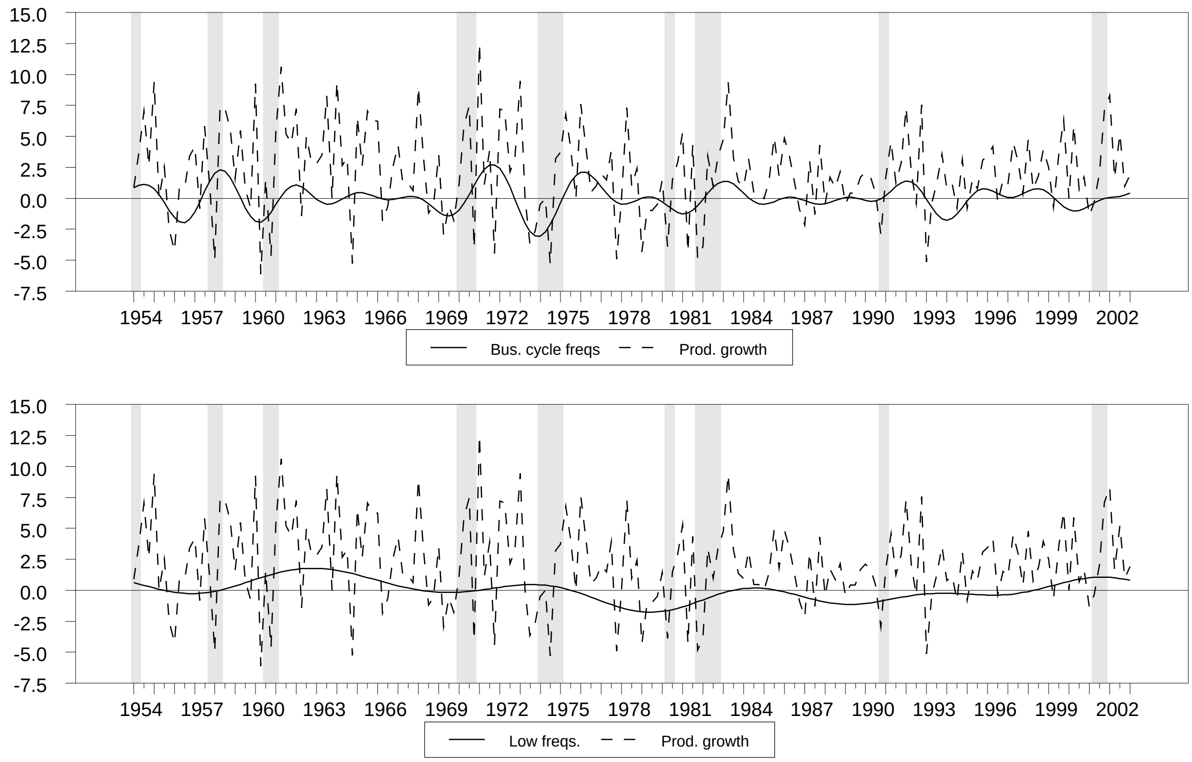
Notes: $y =$ log of output, $n =$ log of hours worked, $x =$ log of labor productivity ($= y - n$), $u =$ unemployment rate, $g =$ log of real GDP, $p =$ log of GDP deflator. Coefficient from regression of in-phase components of the dependent variable and the regressor, with standard errors in parentheses. Number of frequency domain ordinates is 197, cycle is 16.5 quarters, width of spectral smoothing window is 11. See Jenkins and Watts (1968, Section 8.3), Brillinger (1981, Section 8.4), or Engle (1974).

Table 5. Phase lead: number of periods by which variable 1 leads variable 2
1954 Q1 to 2003 Q1

Variable 1	Variable 2 Δx	Δy	Δn	u	Δu
Δy	-2.60 (.96)	–			
Δn	-3.95 (1.14)	-1.01 (.43)	–		
u	.88 (1.08)	3.62 (.60)	4.67 (.63)	–	
Δu	4.26 (1.10)	7.14 (.51)	8.17 (.38)	3.50 (.61)	–
Δg	-2.77 (1.01)	-.16 (.20)	.84 (.46)	-3.81 (.63)	-7.31 (.58)
Δp	6.84 (1.27)	-7.15 (1.53)	-5.90 (1.77)	5.91 (1.22)	2.49 (1.72)

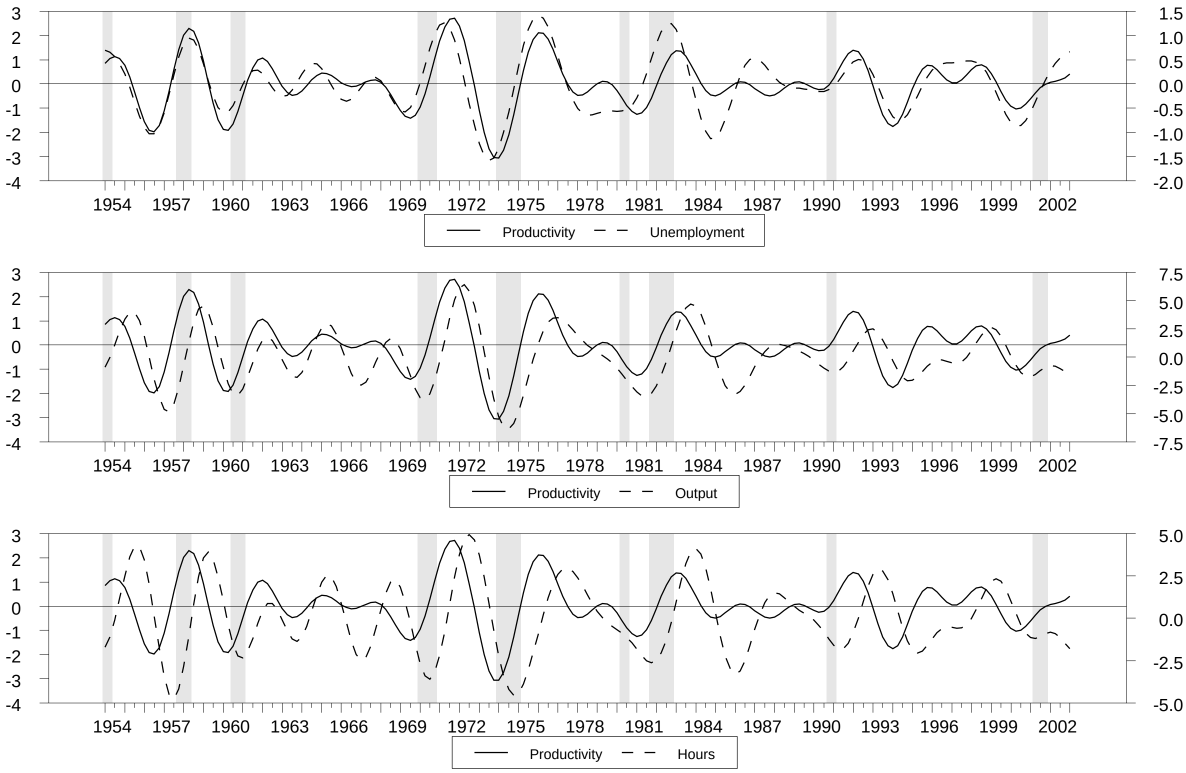
Notes: $y =$ log of output, $n =$ log of hours worked, $x =$ log of labor productivity ($= y - n$), $u =$ unemployment rate, $g =$ log of real GDP, $p =$ log of GDP deflator. Cycle is 16.5 quarters, number of frequency domain ordinates is 197, width of spectral smoothing window is 11. Figures in parentheses are standard errors, obtained as in Brillinger (1981, Section 8.7).

Figure 1. Productivity growth and its business cycle and low-frequency components



Notes: Business cycle and low-frequency components are derived by applying band pass filters in the frequency domain, retaining cycles from 11 to 28 quarters and above 28 quarters, respectively.

Figure 2. Business cycle components of productivity growth, unemployment, output growth, and employment growth



Notes: Business cycle components are derived by applying a band pass filter in the frequency domain, retaining cycles from 11 to 28 quarters. Left scale is for solid line, right scale for dashed line.